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Optimization of Induction Heating Regarding Typical Quality Criteria: Problem Solution Based on 2D FEM Analysis

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Abstract. One of the most widespread methods of heating is induction mass heating because it offers certain advantages over similar technologies, including convectional and electrical heating. A significant economical effect can be achieved through optimization of heating modes and design parameters of induction heaters on the basis of modern optimal control theory for distributed parameters systems. The paper is devoted to the numerical simulation and optimal with respect to typical quality criteria control of thermal modes for metals induction heating before hot forming operations. Two-dimensional non-linear time-optimal control problem, problem of maximum heating accuracy and problem of minimum energy consumption are formulated and reduced to the mathematical programming problems. Optimization procedures are based on the developed at SamSTU alternance method of optimal control theory for distributed parameters systems. 2D FLUX code provides FEM analysis of interrelated electromagnetic and temperature fields during induction heating of a cylindrical billet before its hot forming. The model integrated into optimization procedures provides options for variation of the heating system parameters or billet geometry, and for evaluating the process optimization abilities. Computational results for optimal heating of aluminum cylindrical billets are shown and analyzed.

Introduction

Induction mass heating of metals is one of the most widespread methods of electrical heating before warm and hot forming because it offers certain advantages over competitive technologies such as a controlled reproducible billet temperature, high productivity and reliability, a short heating, an immediate readiness for operation, immediate switch-off when the forging machine is out of operation, and others [1-2]. Processes of metal hot forming with induction-based pre-heating are inseparably linked with the development of key industries and represent very complicated and energy-consuming technologies.

A significant improvement of induction heating system effectiveness can be achieved through optimization of heating modes and design parameters of induction heaters on the basis of modern optimal control theory for distributed parameters systems [3, 4, 5]. Combined application of advanced FEM analysis codes and problem-oriented optimization methods is a powerful approach to develop highly effective induction heating systems.

The presented research is devoted to the numerical simulation and optimization of thermal modes for induction heating with respect to typical quality criteria, such as time-optimal control criterion, maximum heating accuracy and minimum energy consumption.

Statement of 2D non-linear optimal control problems

A space-time temperature distribution within inductively heated cylindrical billet is described by highly complicated system of interrelated Maxwell and Fourier equations for electromagnetic and temperature fields with the appropriate boundary conditions [3]:

$$curl\overline{H} = \sigma(T)\overline{E} + \frac{\partial \overline{D}}{\partial t}; \quad curl\overline{E} = -\frac{\partial \overline{B}}{\partial t}; \quad div\overline{B} = 0; \quad div\overline{E} = 0; \tag{1}$$

$$c(T)\gamma(T)\frac{\partial T(r,l,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda(T)r\frac{\partial T(r,l,t)}{\partial r}\right) + \frac{\partial}{\partial l}\left(\lambda(T)\frac{\partial T(r,l,t)}{\partial l}\right) + W(r,l,t). \tag{2}$$

$$0 < r < R;\ 0 < l < L;\ 0 < t \le t^0$$

Here $\overline{H}$ is vector of magnetic field intensity; $\sigma(T)$ is the electrical conductivity; $\overline{E}$ is vector of electric field intensity; $\overline{D}$ is vector of electric flux density; $\overline{B}$ is vector of magnetic flux density; $T(r,l,t)$ is the temperature distribution varying with time *t,* and radial and axial spatial coordinates *r*, *l,* respectively; R and L are cylinder radius and length; $c(T)$ is specific heat; $\gamma(T)$ is specific density of the metal; $\lambda(T)$ is the thermal conductivity of the metal; $W(r,l,t)$ is internal heat sources power absorbed by surface unit of heated body in time unit.

In the most typical technological processes a maximum admissible value ε of absolute deviation of the final temperature distribution $T(r,l,t^0)$ from the required outlet temperature $T^*(r,l)$ is prescribed. It means that at the end of heating cycle $t = t^0$ the temperature in any point of the billet volume should deviate not more than by value ε from required temperature $T^*(r,l)$:

$$\max_{r\in[0,R];l\in[0,L]} \left|T(r,l,t^0) - T^*(r,l)\right| \le \varepsilon. \tag{3}$$

It will be reasonable to select a voltage of power supply $u(t)$ as a control input with values on the restraint interval:

$$0 \le u(t) \le u_{\max},\ t \in (0;t^0). \tag{4}$$

When maximum productivity is required, a minimal total heating time t^0 can be considered as a cost function written in an integral form as:

$$I_1 = \int_0^{t^0} dt = t^0 \to \min. \tag{5}$$

Among typical cost criteria there is an important cost function which is defined as heating accuracy that can be estimated, according to Eq. 3, by absolute maximum deviation of final temperature distribution $T(r,l,t^0)$ at the end of heating stage from the required one $T^*(r,l)$ within volume V of the heated body. Then given cost criterion I_2 can be defined as:

$$I_2 = \max_{r\in[0,R];l\in[0,L]} \left|T(r,l,t^0) - T^*(r,l)\right| \to \min. \tag{6}$$

The most essential part of induction heating process effectiveness deals with a cost of the consumed time-dependent electrical power $P(t)$. That is why it is often proper to consider a cost function minimizing energy consumption which can be represented in the integral form as:

$$I_3 = \int_0^{t^0} P(t)dt \to \min. \tag{7}$$

The statements of optimal control problem are considered under assumption that complete information about heating process is available and there are no disturbances affecting the heating system and the time of workpiece transfer to hot working equipment is negligible.

Then the statement of optimal control problem can be formulated such as following: it is required to select such control function $u_{opt}(t)$ that provides steering workpiece initial temperature distribution $T(r,l,t)\big|_{t=0} = T_0(r,l)$ to desired temperature $T^*(r,l)$ with prescribed accuracy ε (according to Eq. 3) in minimal optimal process time. Control input is bounded by restriction (Eq. 4), and temperature field is described by Eq. 1-2 with the appropriate boundary conditions.

Maximum heating accuracy or minimum energy consumption problems are formulated by the similar way regarding optimization criteria (Eq. 6 or 7).

Reduction of optimal control problems to mathematical programming problems

It is possible to prove mathematically [6-7] that the solution of the formulated non-linear two-dimensional time-optimal control problem represents the heating process consisting of alternating stages of heating under maximum voltage $u \equiv u_{\max}$ ("Heat ON") and subsequent soaking under $u \equiv 0$ ("Heat OFF") cycles. The number $N \geq 1$ of stages is defined uniquely by given heating accuracy ε and it increases with decreasing ε. Therefore, the shape of optimal control algorithm is known, but the number of stages N and durations $\Delta_1, \Delta_2, \ldots, \Delta_N$ of those stages remain unknown. Therefore, the initial problem is reduced to searching for parameters vector $\mathbf{\Delta} = (\Delta_i),\ i = \overline{1, N}$ $\Delta_i,\ i = 1, 2, \ldots, N$ uniquely specifying optimal control input $u_{opt}(t)$. Now search for control function can be written as:

$$u_{onm}(t) = \frac{u_{\max}}{2}\left[1 + (-1)^{j+1}\right],\ \sum_{i=1}^{j-1} \Delta_i < t < \sum_{i=1}^{j} \Delta_i,\ j = \overline{1, N}. \tag{8}$$

Applying a control input (Eq. 8), at the end of heating process (at $t = t^0 = t_N$) the temperature in any point $T(r,l,t^0)$ depends on values $\Delta_i,\ i = 1, 2, \ldots, N$. This means that the dependence $T(r,l,t^0)$ in Eq. 3 can be replaced by relation $T(r,l,\mathbf{\Delta})$. Thus, condition Eq. 3 required for given final temperature state can be rewritten as:

$$\Phi(\mathbf{\Delta}) = \max_{r \in [0;R];\, l \in [0;L]} \left| T(r,l,\mathbf{\Delta^0}) - T^* \right| \leq \varepsilon_0. \tag{9}$$

The problem now is reduced to determination of such time intervals $\Delta_i,\ i = \overline{1, N}$, of alternating heating and soaking stages, that provide satisfying requirement of Eq. 9 in minimal possible time. Total time is equal to the sum of all Δ_i. Then cost criterion can be determined as a following sum:

$$I_1(\Delta) = \sum_{i=1}^{N} \Delta_i \to \min_{\Delta} \tag{10}$$

From the formal point of view the optimal control problem is reduced to a mathematical programming problem minimizing object function (Eq. 10) of N variables Δ_i, where restraint on a set of admissible values Δ_i is prescribed in the form of Eq. 9.

There is a whole class of optimization problems that can be solved in the manner indicated, namely problems involve the typical cost criteria – Eq. 6-7.

In the problem of maximum absolute heating accuracy for a given time, the optimal control has the same form Eq. 8 as in the time-optimal control problem [3]. Instead of Eq. 9-10 this problem is reduced to the following problem of mathematical programming minimizing cost function $I(\mathbf{\Delta})$ with constraint $\Phi(\mathbf{\Delta})$.

$$I_2(\mathbf{\Delta}) = \max_{r\in[0;R];l\in[0;L]} \left|T(r,l,\mathbf{\Delta}) - T^*\right| \to \min_{\Delta};\ \Phi(\mathbf{\Delta}) = \sum_{i=1}^{N} \Delta_i = \tilde{t}^0. \tag{11}$$

Solution of this problem coincides with the solution of the problem Eq. 9-10. It is proved by mathematical analysis [3] that the control algorithm based on Eq. 8 is optimal with respect to criterion in Eq. 7 under required heating accuracy ε. Power cost is proportional to the sum of odd control intervals instead of the sum Eq. 10 of all control intervals for the time-optimal problem. Then minimum energy consumption problem can be reduced to the following problem of mathematical programming:

$$I_3(\mathbf{\Delta}) = \sum_{i=1,3,5,\ldots,N_1}^{N_1} \Delta_i \to \min_{\mathbf{\Delta}};\quad \varPhi(\mathbf{\Delta}) = \max_{l\in[0;R];y\in[0;L]} \left|T(l,y,\mathbf{\Delta}) - T^*(r,l)\right| \le \varepsilon. \tag{12}$$

Dependencies $T(r,l,\mathbf{\Delta})$ on l, r and $\mathbf{\Delta}$ in the problems can be defined as a result of 2D FEM analysis of induction heating process.

Known formal techniques for solution of mathematical programming problems Eq. 9-10, Eq. 11, Eq. 12 are reduced only to relatively complicated numerical methods; and their general orientation is not connected to the specific content of the problem. An alternative effective approach to such problem solution is alternance method developed by Rapoport [3]. This method offers new tools for design and control of practical, cost-effective induction heating processes due to some advantages over well-known methods. First, it is strongly problem-oriented and takes into account all basic physical features of the optimized system. Second, it allows dramatical reduction for the number of calls of field analysis modules and objective function evaluations. At last, it leads to the global optimal solution that cannot be improved even theoretically.

The alternance method allows constructing the sets of equalities closed in the mathematical sense with respect to all optimized parameters of the heating process. In other words, the number of equalities proves to be equal to the number of all sought parameters that completely define the process under control. This provides potential capability to transform a set of equalities into a set of equations that ought to be solved with respect to unknown parameters, leading to the final solution of optimal control problem. This optimization strategy has been described in many publications [3-7] and applied in this paper to solution of the problems Eq. 9-10, Eq. 11, Eq. 12.

Results of problem solution based on 2D FEM analysis

Let us consider results of problem Eq. 9-10, Eq. 11, Eq. 12 solutions for induction heating of aluminum cylindrical billets up to $450°C$ for the design parameters of induction heating system presented in Table 1.

From the viewpoint of the simulation, the heating process represents a coupled field analysis problem characterized by the interaction of electromagnetic and temperature fields. The finite element 2D model of the heating process has been developed using the commercial software package FLUX. The model allows performing the simulation of the electromagnetic field that induces internal heat sources in the work piece and the consequent transient thermal analysis during the whole heating process. As a result, the model provides the temperature distribution within the work piece as well as the information on electrical parameters of the process.

Electromagnetic and thermal material properties used in the model are temperature dependent and ensure the correct behavior of the model in the whole investigated temperature range. They are evaluated for each time step of the transient thermal analysis. As the arrangement of the work piece and the induction coil can be assumed as axis symmetrical, 2-dimensional modeling has been used for the description of the process. The coupled field analysis problem is solved in the complete longitudinal cross-section of the arrangement containing the coil, the heated work piece, the refractory and the surrounding air. Common material data (relative permeability and electric resistivity) for standard aluminum have been used. For the thermal calculations (thermal conductivity and heat capacity) basic material values of aluminum have been chosen [2].

The results of problem solution are presented in Table 2 for the maximum heating accuracy $\varepsilon = \varepsilon_{\min}^{(1)}$ attainable by one-stage heating under maximum voltage $u \equiv u_{\max}$ (N=1 in Eq. 8), for the maximum heating accuracy $\varepsilon = \varepsilon_{\min}^{(2)}$ attainable by two-stage heating (N=2 in Eq. 8), and for the value of heating accuracy $\varepsilon : \varepsilon_{\min}^{(2)} < \varepsilon < \varepsilon_{\min}^{(1)}$ [7]. As one can see, in accordance with the physical regularities the process with a better heating accuracy requires higher energy consumption. At the same time the solutions for the value $\varepsilon = \varepsilon_{\min}^{(2)}$ to all problems coincide.

Table 1. Design parameters of induction heating system.

Number of turns	69	Length of the billet [m]	1.0
Geometry of turn [m]	0.016x0.012x0.002	Distance between billet and inductor [m]	0.05
Distance between turns [m]	0.004	Frequency [Hz]	50
Diameter of the billet [m]	0.5	Maximum voltage [V]	470

Table 2. Results of problem solutions.

$\varepsilon_{\min}^{(2)} < \varepsilon < \varepsilon_{\min}^{(1)}$	Results of solution for time-optimal and maximum heating accuracy problems				Results of solution for minimum energy consumption problem			
	$\varepsilon, {}^0C$	Δ_1^0,sec	Δ_2^0,sec	E, MJ	$\varepsilon, {}^0C$	Δ_1^0,sec	Δ_2^0,sec	E, MJ
$\varepsilon = \varepsilon_{\min}^{(1)}$	85	1552	-	228,8	85	1430	179	207,9
$\varepsilon_{\min}^{(2)} < \varepsilon < \varepsilon_{\min}^{(1)}$	50,8	1640	45	243,4	39,1	1610	164	238,29
$\varepsilon = \varepsilon_{\min}^{(2)}$	25,4	1665	158	247,67	25,4	1665	158	247,67

Fig. 1 represents the final temperature distribution within billet volume for the heating accuracy $\varepsilon = \varepsilon_{\min}^{(2)}$ when control algorithm is optimal regarding all three considered cost functions [7].

Conclusions

Two-dimensional nonlinear optimal control problems have been mathematically formulated in terms of the typical optimization criteria, e.g. minimum heating time, maximum heating accuracy and minimum energy consumption.

The problems have been reduced to the problems of mathematical programming and solved on the basis of alternance method of parametric optimization. Implementing the combination of a FEM code with and optimization procedure based on the alternance method makes the proposed approach specific and original providing optimal control of induction heating processes.

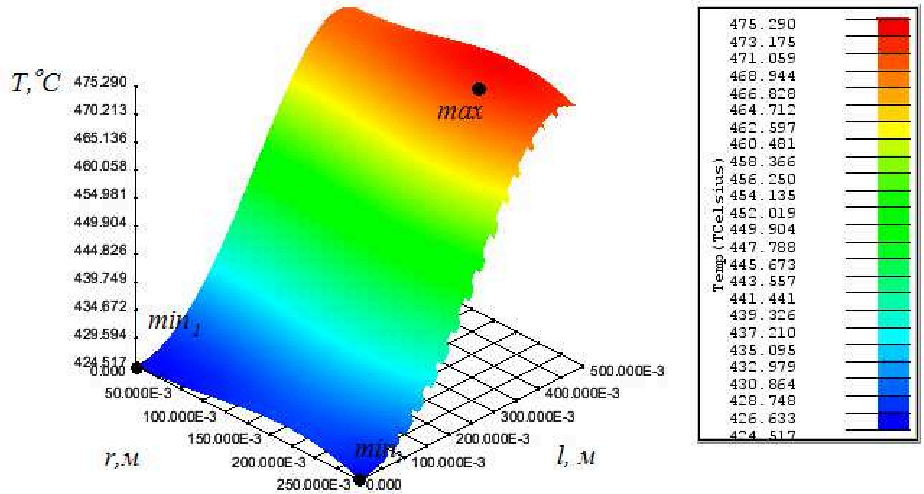

Fig. 1. Temperature field within the heated billet' volume at the end of heating process for $\varepsilon = \varepsilon_{\min}^{(2)}$.

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References

[1] V. Rudnev, D. Loveless, R. Cook, M. Black, Handbook of Induction Heating, Marcel Dekker Inc., New York, USA, 2006.

[2] Yu. E Pleshivtseva, E.Ya. Rapoport, Joint optimization of interrelated thermophysical processes in metal working systems based on system quality criteria, Optoelectronics, Instrumentation and Data Processing. 49(6) (2013) 527-535.

[3] E. Rapoport, Yu. Pleshivtseva, Optimal Control of Induction Heating Processes, CRC Press/Taylor & Francis Group, New York, USA, 2007.

[4] Yu. Pleshivtseva, E. Rapoport, The successive parameterization method of control actions in boundary value optimal control problems for distributed parameter systems, Journal of Computer and Systems Sciences International. 48, 3 (2009) 351-362.

[5] Yu. Pleshivtseva, and E. Rapoport, Algorithmically precise method of parametric optimization in boundary-value optimal control problems for distributed parameter systems, Optoelectronics, Instrumentation and Data Processing. 45, 5 (2009) 464-471.

[6] E. Rapoport, Yu. Pleshivtseva, Optimal control of nonlinear objects of thermophysics engineering, Optoelectronics, Instrumentation and Data Processing. 48, 5 (2012) 429-437.

[7] Yu. Pleshivtseva, A. Dyakonov, A. Popov, Two-dimensional problem of optimal with respect to typical quality criteria control of induction heating, Bulletin of Samara State Technical University, Issue Technical Sciences. 2(42) (2014) 148-163.